MATRIX OF TRANSFER COEFFICIENTS OF A MULTIELEMENT PLASMA AND ITS APPLICATION IN PROBLEMS OF HIGH-TEMPERATURE GASDYNAMICS

G. A. Pavlov

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Investigations of flow and processes of heat and mass exchange of a multielement plasma with confining surfaces are required in the construction of prospective power plants and other engineering devices [1]. The experimental study of heat and mass change in these devices is difficult, so one resorts to numerical modeling of the motion of a multicomponent, partially ionized plasma in the approximation of local thermodynamic equilibrium (LTE) on the basis of a certain system of equations (see, e.g., [2]) with assigned boundary and initial conditions. The range of parameters in which one must assign the thermophysical properties of a substance for numerical modeling ($p \approx 10^{-1}$-10^{3} MPa, $T \approx 10^{3}$-10^{5}°K) includes the important and inadequately studied case of a plasma with strong Coulomb interaction. A systematic description of the properties of such a plasma is impossible owing to the strong interparticle interaction, so that the problem arose of constructing theoretical models based, in particular, on the known experimental data.

Kucherenko and Pavlov [3, 4] formulated a model approach to the calculation of the kinetic coefficients of a nonideal plasma: the viscosity coefficients ζ and η, the transport thermal conductivity λ', and the multicomponent coefficients of diffusion D_{ik} and thermodiffusion D_i^t. For this purpose it is proposed to use the system of classic kinetic equations, the collision integrals of which are defined in the Boltzmann form with allowance for the elementary processes important in a nonideal plasma, the qualitative properties of its composition, and data on the kinetic coefficients of nonideal classic Coulomb systems. Such an approach allows us to separate the contributions to the kinetic coefficients due to the concrete composition of the plasma and to the strong Coulomb interaction in it, i.e., the non-Coulomb and Coulomb effects. In the transition from D_{ik} and D_i^t to the effective transfer coefficients (ETC), through which the mass fluxes of chemical elements J_a (including the electric current J_e) and the heat flux J_q are expressed, we must, according to [3, 4], take into account the nonideality in the thermodynamic forces.

The above scheme of calculation of the ETC is rather complicated. Therefore, as a control on the numerical values of the ETC (ζ, η, $\lambda' > 0$) it is necessary to use general restrictions on the nonlinear nondiagonal ETC matrix describing the transfer of energy and mass (charge) in a multielement plasma in the LTE approximation. The properties of the ETC matrix are also important when the latter is used in problems of high-temperature gasdynamics. The elements of the ETC matrix are connected with the coefficients to the leading derivatives (but do not coincide with them) in the system of equations of diffusion of the chemical elements and energy. Obviously, the character of the solutions of the system of equations of diffusion and energy depend on the properties of the matrix coefficients to the leading derivatives in this system. We shall investigate the matrix of coefficients to the leading derivatives in the system of equations of diffusion of chemical elements and energy, as well as the ETC matrix, starting from the equations of thermodynamics of irreversible processes, formulated relative to the chemical potentials of the elements, and from the conditions of thermodynamic stability.

The equations of diffusion of the chemical elements and energy, under the condition that E, H = 0, p = const, and $\mathbf{v} = 0$ (emission is not taken into account), have the form [2]

$$\rho dc_a/dt = -\operatorname{div} \mathbf{J}_a + \dot{c}_a,\ 1 \leqslant a \leqslant N_a - 1,\ \rho dh/dt = -\operatorname{div} \mathbf{J}_q + \dot{h}, \tag{1}$$

where ρ is the density; $\dot{c}_a$ and $\dot{h}$ are sources; h is the specific enthalpy of the plasma; $c_a = \Sigma u_{ia} c_i m_a/m_i$; $c_i = \rho_i \Big/ \sum_{k=1}^{N} \rho_k$; $\rho_i = m_i n_i$; n_i and m_i are the concentration and mass of component i; N is the number of components in the plasma; u_{ia} is the number of nuclei of element a in

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component i; m_α is the mass of element α; N_α is the number of chemical elements forming the plasma; J_α and J_q have the form

$$\mathbf{J}_a = \sum_{b=1}^{N_a-1} D_{ab}\nabla c_b + D_a^t \nabla T, \quad \mathbf{J}_q = -(\lambda' - \lambda^t)\nabla T + \sum_{a=1}^{N_a-1} \lambda_a \nabla c_a, \tag{2}$$

where $D_{\alpha b}$ and D_α^t are the effective coefficients of diffusion and thermodiffusion; λ^t and λ_α are the effective coefficient of thermal conductivity and the effective diffusional thermal coefficient. Since $h = h(p, T, c_1 \ldots c_{N_\alpha - 1})$ and J_α and J_q are assigned in the form (2), we write (1) in the matrix form

$$\frac{\partial u}{\partial t} = \operatorname{div}(a(u)\nabla u) + \ldots, \quad u = \begin{pmatrix} c_1 \\ \vdots \\ c_{N_a-1} \\ T/T_\delta \end{pmatrix}, \tag{3}$$

$$a(u) = \begin{pmatrix} \rho & & & \\ 0 & \rho & \ddots & 0 \\ \vdots & & \ddots & \\ \frac{\rho c_{c_1}}{\lambda} & \frac{\rho c_{c_2}}{\lambda} & \ldots & \frac{T_\delta \rho c_p}{\lambda} \end{pmatrix}^{-1} \begin{pmatrix} D_{11} & -D_{12} & \ldots & -T_\delta D_1^T \\ -D_{21} & -D_{22} & \ldots & -T_\delta D_2^T \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\lambda_1}{\lambda} & -\frac{\lambda_2}{\lambda} & \ldots & \frac{T_\delta(\lambda' - \lambda^T)}{\lambda} \end{pmatrix},$$

where $c_p = (\partial h/\partial T)_{p,\{c_\alpha\}}$; $c_{c_i} = (\partial h/\partial c_i)_{p,T,c_\alpha \neq c_i}$, $T\delta$, $\lambda > 0$ are normalization coefficients; the first matrix in the expression for $a(u)$ [we designate it as $b^{-1}(u)$] is determined by the thermodynamic properties of the plasma and the second [$a'(u)$] is the ETC matrix; the terms to the lowest derivatives are omitted here. We note that $a(u)$ is not a normal matrix, i.e., $a(u)\tilde{a}(u) \neq \tilde{a}(u)a(u)$; thus, $a(u)$ is not made diagonal through a unitary transformation [5]. Moreover, the matrix $a(u)$ is essentially nonlinear, since $D_\alpha^t(p, T; c_\alpha = 0.1) = 0$. In the investigation of solutions of (3) with definite boundary and initial conditions (see, e.g., [6-8]) the matrix $a(u)$ has traditionally been assumed to be parabolic (i.e., the eigenvalues of the matrix must have positive real parts). This condition guarantees that the solutions of (3) are continuous and positive [8]. Let us investigate the parabolic nature of the matrices $a'(u)$ and $a(u)$. We express $a'(u)$ through the phenomenologic kinetic coefficients and thermodynamic derivatives, using the formulation of the entropy production $\dot{s}$ relative to the chemical potentials of the elements. In this case we write $\dot{s}$ in the form (see [2])

$$\dot{s} = -\frac{1}{T^2}\mathbf{J}_q' \nabla T - \frac{1}{T}\sum_{a=1}^{N_a-1} \mathbf{J}_a \nabla_T(\mu_a - \mu_e), \tag{4}$$

where $\mathbf{J}_q' = \mathbf{J}_q - \sum_{a=1}^{N_a} h_a \mathbf{J}_a$; μ_α (μ_e) and h_α (h_e) are the chemical potentials and specific enthalpies of the chemical elements (space charge), respectively; $h = \sum_{a=1}^{N_a-1} c_a h_a$; $\mu_a = (\partial E/\partial c_a)_{p,s,c_b \neq c_a}$; $h_\alpha = \mu_\alpha + Ts_\alpha$; $s_\alpha = -(\partial F/\partial T)_{\rho,\{c_\alpha\}}$; $F = E - Ts$, E is the specific internal energy of the plasma. By the Curie principle

$$\begin{aligned} \mathbf{J}_a &= -\alpha_{aQ}\frac{\nabla T}{T^2} - \frac{1}{T}\sum_{b=1}^{N_a-1} \alpha_{ab}\nabla_T(\mu_b - \mu_e), \\ \mathbf{J}_q' &= -\alpha_{QQ}\frac{\nabla T}{T^2} - \frac{1}{T}\sum_{b=1}^{N_a-1} \alpha_{Qa}\nabla_T(\mu_a - \mu_e) \end{aligned} \tag{5}$$

(α_{ij} are phenomenological kinetic coefficients). To write J_α and J_q in the form (2), i.e., to express the ETC through α_{ij}, we use the dependence $\mu_\alpha = \mu_\alpha(p, T, \{c_\alpha\})$ and the connection between J_q and J_q', and then

$$\begin{aligned} D_{ab} &= -\frac{1}{T}\sum_{l=1}^{N_a-1} \alpha_{al}\mu_b^l, \quad D_a^t = -\frac{\alpha_{aQ}}{T^2}, \\ \lambda_a &= \sum_{b=1}^{N_a-1}\left[-\frac{\alpha_{Qb}}{T}\mu_a^b + (h_b - h_e)D_{ba}\right], \quad \tilde{\lambda}^t = \sum_{a=1}^{N_a-1}(h_a - h_e)D_a^t. \end{aligned} \tag{6}$$

Here $\mu_b^a = [\partial(\mu_a - \mu_e)/\partial c_b]_{p,T,c_a \neq c_b}$; $(\alpha_{Qb}/T)\,\mu_a^b \equiv \lambda_a'$. We note that $\tilde{\lambda}^t$ does not coincide with λ^t, since J_q is written through the plasma components in the definition of λ^t. In exactly the same way $\alpha_{QQ}/T^2 \equiv \lambda \neq \lambda'$, but, obviously $\lambda' - \lambda^t = \lambda - \tilde{\lambda}^t$ and $\tilde{\lambda} = \lambda' + \lambda_R > 0$, where λ_R is the so-called "chemical" thermal conductivity of the plasma [2]. The matrix form of Eqs. (6), i.e., the expressions for the ETC, is (with the opposite sign)

$$\begin{pmatrix} 1 & & & \\ 0 & & 1 & 0 \\ \vdots & & \ddots & \\ -\frac{h_1 - h_e}{\lambda} & -\frac{h_2 - h_e}{\lambda} & \cdots & \frac{T}{T_\delta} \end{pmatrix} \left[\begin{pmatrix} \frac{\alpha_{11}}{T} & \frac{\alpha_{12}}{T} & \cdots & \frac{T_\delta}{T^2}\alpha_{1Q} \\ \frac{\alpha_{21}}{T} & \frac{\alpha_{22}}{T} & \cdots & \frac{T_\delta}{T^2}\alpha_{2Q} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{T_\delta}{T^2}\alpha_{Q1} & \frac{T_\delta}{T^2}\alpha_{Q2} & \cdots & \frac{T_\delta^2\tilde{\lambda}}{T\lambda^2} \end{pmatrix} \begin{pmatrix} \mu_1^1 & \mu_2^1 & \cdots & 0 \\ \mu_1^2 & \mu_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & & \lambda \end{pmatrix} \right]. \tag{7}$$

The matrix inside the brackets is parabolic, since it is the product of two positive-definite matrices α and μ. The matrix $-D$, which is obtained from the matrix $a'(u)$ without the last row and last column, is also parabolic and determines the matrix to the leading derivatives in the system of equations of diffusion of the chemical elements (without baro- and thermodiffusion) [9]. The matrix $a'(u)$ given by expression (7) is not parabolic, generally speaking.

We rewrite $a(u)$ in a form more convenient than (3):

$$a(u) = \begin{pmatrix} 1/\rho & & & \\ 0 & & 1/\rho & 0 \\ \vdots & & \ddots & \\ \frac{(h_1 - h_e)_{c_1}'}{\rho c_p T_\delta} & \frac{(h_2 - h_e)_{c_2}'}{\rho c_p T_\delta} & \cdots & \frac{T}{T_\delta^2}\frac{\lambda}{\rho c_p} \end{pmatrix} \alpha\mu. \tag{8}$$

The first matrix on the right side of (8) is diagonal in the "ideal" case, when $(h_\alpha - h_e)$ does not depend on $\{c_\alpha\}$. Since the matrix inside the brackets in (7) is parabolic, in the "ideal" case $a(u)$ is also parabolic, while in the "nonideal" case [$(h_\alpha - h_e)$ depends on $\{c_\alpha\}$] $a(u)$ is nonparabolic, generally speaking. As an example, let us consider the eigenvalues of $a(u)$ for a two-element medium. In this case the matrix to the leading derivatives in the system (3) and its eigenvalues are

$$a(u) = \begin{pmatrix} -D_{aa}/\rho & -T_\delta D_a^t/\rho \\ \frac{c_c D_{aa}}{\rho T_\delta c_p} - \frac{\lambda_a}{\rho T_\delta c_p} & \frac{c_c T_\delta D_a^t}{\rho T_\delta c_p} + \frac{T_\delta(\lambda' - \lambda^t)}{\rho T_\delta c_p} \end{pmatrix},$$

$$\lambda_{1,2} = -\frac{D_{aa}}{2\rho} + \frac{c_c D_a^t}{2\rho c_p} + \frac{\lambda' - \lambda^t}{2\rho c_p} \pm \tag{9}$$

$$\pm\frac{1}{2}\sqrt{d^2 + 4\frac{D_{aa}}{\rho^2 c_p}\left((h_a - h_e)_c' c D_a^t + \tilde{\lambda}\right) + 4\frac{D_a^t}{\rho^2 c_p}(-c_c D_{aa} + \lambda_a)},$$

where $-d$ is twice the expression in front of the square root in the equation for $\lambda_{1,2}$. Using (6) it is easy to show that the sign of d is not definite in the "nonideal" case, while $d < 0$ in the "ideal" case. The expression inside the square root is

$$D_{aa}\left((h_a - h_e)_c' c D_a^t + \tilde{\lambda}\right) + D_a^t(-c_c D_{aa} + \lambda_a) = D_{aa}\tilde{\lambda} +$$
$$+ D_a^t\left(-(h_a - h_e) D_{aa} - \lambda_a' + (h_a - h_e) D_{aa}\right) = D_{aa}\tilde{\lambda} - D_a^t\lambda_a' < 0$$

and the properties of the matrix $a(u)$ in (9) are determined mainly by the sign of d.

Thus, we investigated the properties of the nonlinear and nondiagonal matrix to the leading derivatives in the system of equations of diffusion and energy for a multielement plasma in an ideal multielement plasma. In a nonideal multielement plasma $a(u)$ is nonparabolic, generally speaking, and one must make a combined consistent calculation of the transfer and thermodynamic characteristics to determine the properties of the matrix to the leading derivatives, i.e., the sign of the expression $-D_{aa}/2\rho + c_c D_a^t/2\rho c_p + (\lambda' - \lambda^t)/2\rho c_p$ in (9), for example. One must also clarify the gasdynamic consequences of the possible nonparabolic nature of $a(u)$.

LITERATURE CITED

1. V. M. Ievlev, "Some results on investigation of a gas-phase, cavity-type nuclear reactor," Izv. Akad. Nauk SSSR, Energ. Transp., No. 6 (1977).
2. V. M. Ievlev (ed.), Thermophysical Properties of the Working Media of a Gas-Phase Nuclear Reactor [in Russian], Atomizdat, Moscow (1980).
3. G. A. Pavlov, "Transfer coefficients of a plasma with strong Coulomb interaction," Zh. Tekh. Fiz., No. 5 (1984).
4. V. I. Kucherenko and G. A. Pavlov, "Calculation of the effective coefficients of thermodiffusion of a nonideal multielement plasma," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1978).
5. G. A. Korn and T. M. Korn, Manual of Mathematics, McGraw-Hill, New York (1967).
6. A. A. Shiryaev and G. A. Pavlov, "Hypersonic flow over a spherical probe in the atmosphere of Jupiter," Preprint Ob'ed. Inst. Khim. Fiz. Akad. Nauk SSSR (1982).
7. G. A. Pavlov and A. A. Shiryaev, "Dissipative structures in a plasma with volumetric heat release," Pis'ma Zh. Tekh. Fiz., 9, No. 21 (1983).
8. A. I. Vol'pert and R. S. Tishakova, "Positive solutions of the second boundary-value Problem for quasilinear parabolic equations," Preprint Ob'ed. Inst. Khim. Fiz. Akad. Nauk SSSR (1981).
9. P. K. Gupta and A. R. Cooper, "The [D]-matrix for multicomponent diffusion," Physica, 54, No. 1 (1971).

SUPERHEATED-IONIZATION INSTABILITY OF AN EXTERNALLY MAINTAINED DISCHARGE

N. M. Maslennikov

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A large number of papers have been devoted to instabilities of an externally maintained discharge, which is quite proper, since they limit the maximum current and voltage on the discharge gap. Instabilities are initiated by processes both in the cathode region and in the volume of the discharge gap. The experimental results presented in [1-3] can serve as an example of the latter case. Typical current oscillograms illustrating the development of instability in nitrogen in a pulsed regime are presented in Fig. 1, based on data of these papers. The higher the voltage U on the discharge gap, the greater the current density j of the discharge and the smaller the time τ of development of instability. The interpretation of these experiments has undergone considerable changes in recent years. The authors of [1-3] initially eliminated the possibility of gas heating during the action of the voltage pulse, since it was assumed that the Joule energy released in the volume of the discharge gap goes almost entirely into the excitation of vibrational degrees of freedom of nitrogen molecules, while the observed instability was explained by step-by-step ionization. After the publication of [4, 5], it was recognized in [6] that a certain fraction of the Joule energy is indeed expended on heating of the nitrogen.

The authors of [7] also came to the conclusion that under the experimental conditions of [1] instability can develop only through step-by-step ionization, and the calculated and measured times of development of instability are close for $E/N > 3.5 \cdot 10^{-16}$ V·cm^2. The considerable discrepancies at lower values of the ratio E/N were explained by neglect of the field nonuniformity.

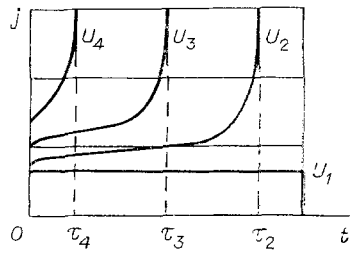

Fig. 1

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